

Introduction to International Perspectives on Problem Solving Research in Mathematics Education

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Any field of research and innovation must be exposed to revisions, criticisms and to an intense scrutiny not only to discuss the state of the art but, hopefully, to identify prospective changes and new areas of study and exploration as well.

Problem Solving has been such an area, with a prominent place in mathematics education and whose contributions continuously appear in conference proceedings, handbooks, journals, books and, more recently, in digital endeavors. Problem Solving involves an approach that fosters reflection and delving into mathematical ideas to explain individuals' cognitive behaviors within social media. Here, we argue that ideas do not live by themselves isolated from the semantic networks that sustain the life of cognition: meaning. These networks constitute a key ingredient for developing understanding and structural perspective of concepts through problems. In the long term, (and maybe not that long) these networks provide integration of knowledge that learners need to construct and integrate in order to gain a wide perspective.

Problem Solving drives developments through research programs, curriculum design, teachers' mathematical education, and mathematical instruction at the level of the classroom. Taking this and more into due account, and in order to identify current trends in mathematical problem solving and to foster a further exchange of ideas within our

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community, we discussed the present project with Bharath Sriraman, the editor of *The Mathematics Enthusiast*, who generously accepted to devote a special issue of the *Journal to Mathematical Problem Solving*. We invited colleagues, who have made significant contributions to this field, to contribute to the special issue. Previously, we had identified some lines of development to eventually frame their contributions. Of course this was not meant as a restriction to their freedom; rather to orient their possible directions. Some questions were posed and discussed to help us identify possible themes to consider in the volume. Thus, we tried to answer: *What are the current trends in problem solving research and what are the main results that influence teachers' practices and curricula design?* In addition, with the significant development of digital tools and environments, we are in need to understand to what extent the present research agenda is being or need to be transformed by their influence. This touches, for instance, deep epistemic issues concerning the nature of valid mathematical reasoning and results in mathematics in the classroom. We have to take into account that mediation tools are not neutral, neither from the cognitive nor from the epistemic viewpoint. That the knowledge students generate and/or appropriate, is intertwined with these tools. However, we cannot forget that a school culture always leaves significant marks on students' and teachers' values. Artigue (2005, p. 246) states that "these [previous] values were established, through history, in environments poor in technology, and they have only slowly come to terms with the evolution of mathematical practice linked to technological evolution." Thus, the school culture requires the gradual re-orientation of its practices to gain access to new habits of mind and to the new environments resulting from a serious presence of digital technologies.

We consider that how we understand the learning of mathematics through a problem-solving approach is deeply related, today, with the presence of the mediation tools that students will find and use in and outside the classroom.

We have shared with the invited authors a list of themes; we would like to mention some of them we consider particularly relevant.

Mathematical Problem Solving Foundations. Any domain of study needs to make explicit tenets and principles that support and justify its academic agenda. As we mentioned before, we are interested in documenting the extent to which theoretical and pragmatic frameworks are helpful to explain the problem solvers' development of new mathematical knowledge. Besides taking into account the role of mediational tools as the foundation of a research program, we need to consider as well the contrasts with a modeling approach to problem solving.

It is relevant to investigate how the presence of digital tools has transformed the agenda of problem solving approaches, which, in its early stages, has developed within a culture of paper and pencil mathematics. Of course, the lines of reasoning supported with and within, the new expressive media reflects what we have, before, termed the cognitive and epistemological consequences of the digital tools.

Mathematical Problem Solving and International Students' Mathematical Assessment. Currently, results from international assessments like PISA or TIMSS are used to compare or contrast students' mathematical competences among different

countries. In general, the media use the results to talk about the success or failure of national educational systems in science, mathematics, and language. Thus, it becomes important to discuss issues regarding the role of problem solving activities in the students' development of competences associated with those types of assessments. Some questions to discuss in this section involve:

- a) How are PISA and TIMSS goals and ways to assess students' mathematical achievement related to mathematical problem solving? Is the PISA framework consistent with frameworks used in mathematical problem solving?
- b) What makes a good task or problem foster and evaluate the students' mathematical thinking? The role of routine and non-routine problems in problem solving approaches.
- c) How can a problem be used for teaching and evaluating the students' comprehension of mathematical concepts?
- d) To what extent should we expect students to pose and solve their own problems?
- e) To what extent international assessments like PISA or TIMSS actually evaluate problem-solving competences, including those that demand the use of computational technology?

Mathematical Problem Solving and Curriculum Frameworks. A distinguishing feature of some current curriculum proposals is that they are structured to enhance problem-solving activities through all school levels. However, there is a need to discuss

what those proposals entail and should include in terms of contents and mathematical processes. Thus, relevant questions to discuss in this section involve the structure and organization of a curriculum centered in problem solving activities. Besides, we need to identify fundamental ideas and processes that are central to foster students' appropriation of mathematical knowledge and the ways digital media can be incorporated within the eventual proposals. Needless to say, the assessments conundrum will be lurking turning the corner.

As a consequence, the presence of digital technologies in education calls us to address this fundamental issue that curricular structures eventually will be inhabited by these technologies. It has already happened in the past: the technology of writing and the technology of positional notation of numbers are two of the milestones in the history of semiotic representations with a living impact on education.

Future developments of Mathematical Problem Solving. It is widely recognized that students should develop abilities, mathematical resources, and ways of thinking that help them formulate and solve not only school problems but also situations that they encounter outside the institutional settings. In this context, it becomes important for students to examine and explore phenomena in which they have the opportunity to examine information embedded in a variety of contexts in order to formulate, explore, and formulate meaningful mathematical questions. Thus, we will be in need to research the extent to which students can transfer problem-solving experiences learnt within the school context to new situations. This will of course, demands from the students the

abilities to move across the semantic field of a mathematical notion. This is far from being a trivial activity.

For instance how could students through problem solving phases that involve gathering data, modeling activities, find solutions and provide interpretations?

We received a positive response to our invitation letter from the authors and their contributions often address several issues discussed above. We hope that readers will find the contents of these two special issues useful to reflect on and extend their views about problem solving and we invite all to continue the discussion directly with the authors and other members of the problem solving community.

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